

Chapter 2: Polynomial and Rational Functions

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Chapter 2: Polynomial and Rational Functions
Topic 1: Complex Numbers

What is an imaginary number? What is a complex number?

The imaginary unit is defined as $i = \sqrt{-1}$

A **complex number** is defined as the set of all numbers in the form of $a + bi$, where a is the real component and b is the coefficient of the imaginary component.

An **imaginary number** is when the real component (a) is zero.

Checkpoint: Since $i = \sqrt{-1}$

Then $i^2 =$

Operations with Complex Numbers

Adding & Subtracting: Combine like terms

$$(a + bi) + (c + di) = (a + c) + (b + d)i$$

Examples:

1. $(5 - 11i) + (7 + 4i)$

2. $(-5 + 7i) - (-11 - 6i)$

3. $(5 - 2i) + (3 + 3i)$

4. $(2 + 6i) - (12 - 4i)$

Multiplying: Just like polynomials, use the distributive property. Then, combine like terms and simplify powers of i .

Remember! Multiplication does not require like terms. Every term gets distributed to every term.

Examples:

1. $4i(3 - 5i)$

2. $(7 - 3i)(-2 - 5i)$

3. $7i(2 - 9i)$

4. $(5 + 4i)(6 - 7i)$

5. $(3 + 5i)(3 - 5i)$

6. $3i(2 - 3i)^2$

A note about conjugates: Recall that when multiplying conjugates, the middle terms will cancel out. With complex numbers, this becomes even simpler:

$$(a + bi)(a - bi) = a^2 + b^2$$

Try again with the shortcut: $(3 + 5i)(3 - 5i)$

Dividing: Just like polynomials and rational expressions, the denominator must be a rational number. Since complex numbers include imaginary components, these are not rational numbers. To remove a complex number from the denominator, we multiply numerator and denominator by the conjugate of the denominator.

Remember! You can simplify first IF factors can be canceled. NO breaking up terms.

Examples:

1. $\frac{7+4i}{2-5i}$

2. $\frac{5+4i}{4-2i}$

Operations with Square Roots of Negative Numbers

Begin by expressing all square roots of negatives in terms of i , then proceed with the operation

Examples:

1. $\sqrt{-18} - \sqrt{-8}$

2. $\frac{-14 + \sqrt{-12}}{2}$

3. $(-1 + \sqrt{-5})^2$

4. $(-2 + \sqrt{-3})^2$

5. $\sqrt{-27} + \sqrt{-48}$

6. $\frac{-25 + \sqrt{-50}}{15}$

Homework

Write all results in Standard Form.

1) $(7 + 2i) + (1 - 4i)$

2) $(-2 + 6i) + (4 - i)$

3) $6 - (-5 + 4i) - (-13 - i)$

4) $7 - (-9 + 2i) - (-17 - i)$

5) $(-5 + 4i)(3 + i)$

6) $(5 - 2i)^2$

7) $\frac{3}{4 + i}$

8) $\frac{3 - 4i}{4 + 3i}$

9) $(-3 - \sqrt{-7})^2$

10) $\frac{-12 + \sqrt{-28}}{32}$

Chapter 2: Polynomial and Rational Functions
Topic 2: Quadratic Functions (Day 1)

Do Now: Solve by completing the square. Use your calculator to check your answers

1. $x^2 + 14x - 15 = 0$

2. $k^2 + 23 = 12k$

3. $b^2 + 2b = -20$

4. $a^2 = -3 + 4a$

Graphing Quadratic Functions

To graph quadratic functions we look for 4 key features:

1. Does it open up or down?

- A leading coefficient (a) that is positive will cause the parabola to open up.
- A leading coefficient (a) that is negative will cause the parabola to open down.

2. What is the VERTEX of the parabola?

- In standard form: Vertex = (h, k)
- In quadratic form: Vertex = $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$ or put it in standard form.

3. What are the x-intercepts?

- Solve by setting the equation equal to zero ($f(x) = 0$)

4. What is the y-intercept?

- Solve by evaluating at zero ($f(0) =$)

Then, plot all of the key features, and sketch a smooth parabola.

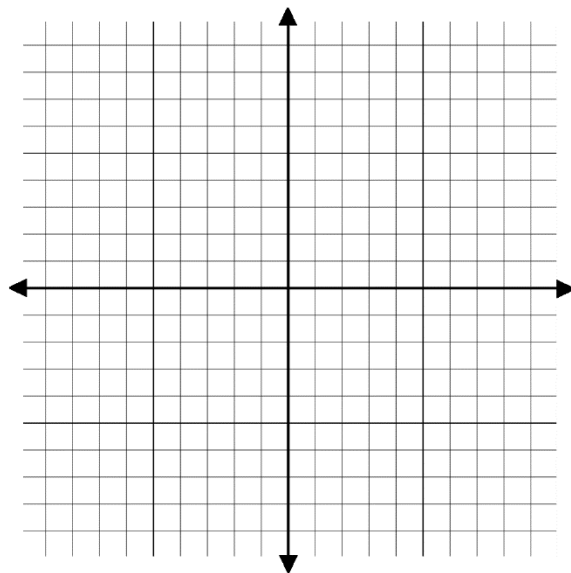
Finally, draw a dotted line for the axis of symmetry.

Type 1: Standard Form $f(x) = a(x - h)^2 + k, a \neq 0$

Example: Graph the quadratic function $f(x) = -2(x - 3)^2 + 8$

1. Up or down?
2. Vertex:
3. $f(x) = 0$

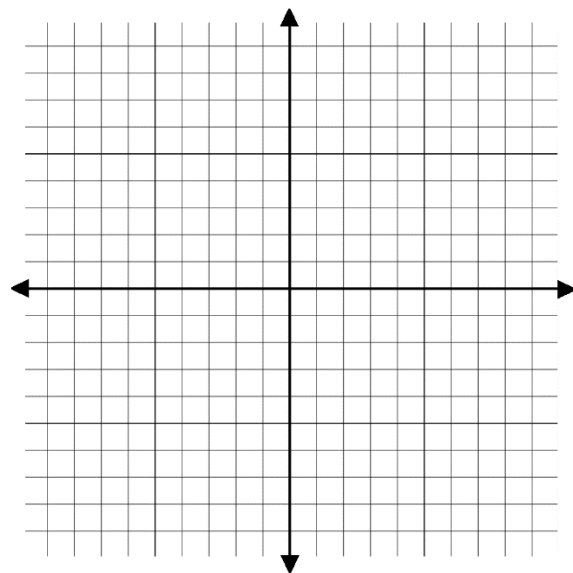
4. $f(0) =$



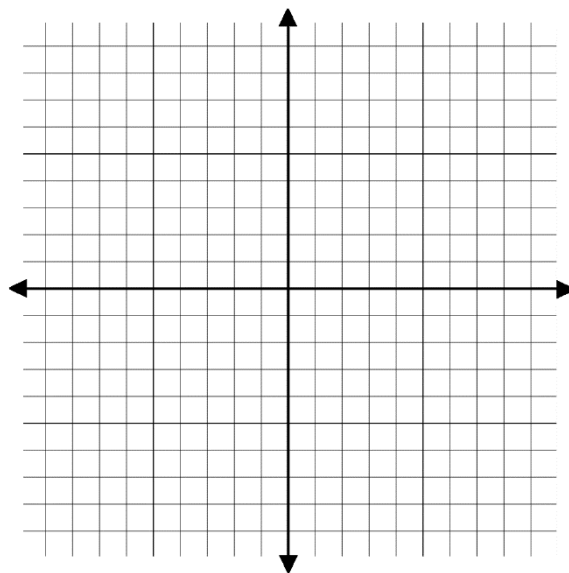
Plot, Sketch, Dot, Label

Your Turn:

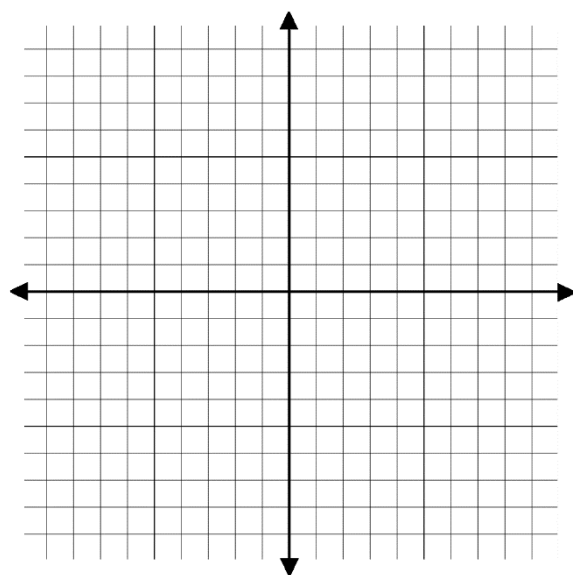
1. $f(x) = (x + 3)^2 + 1$



2. $f(x) = -(x - 1)^2 + 4$



3. $f(x) = (x - 2)^2 + 1$



Type 2: Quadratic Form $f(x) = ax^2 + bx + c, \quad a \neq 0$

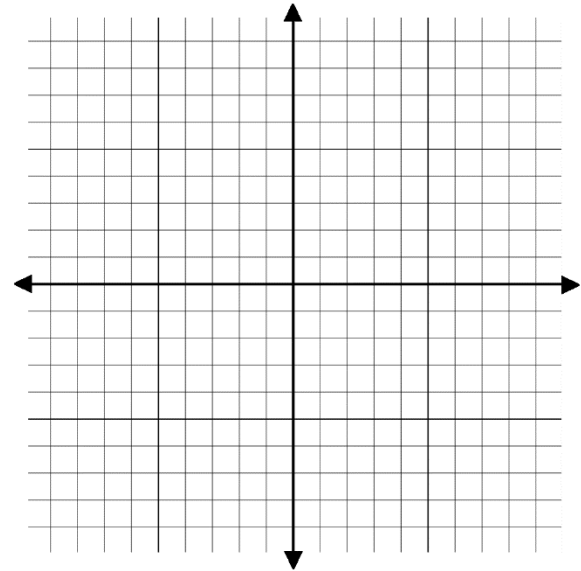
Example: Graph the quadratic function $f(x) = -x^2 + 4x - 1$

1. Up or down?

2. Vertex:

3. $f(x) = 0$

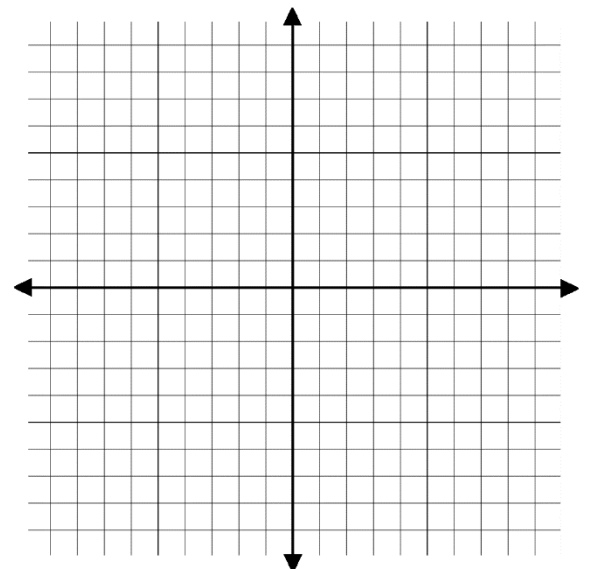
4. $f(0) =$



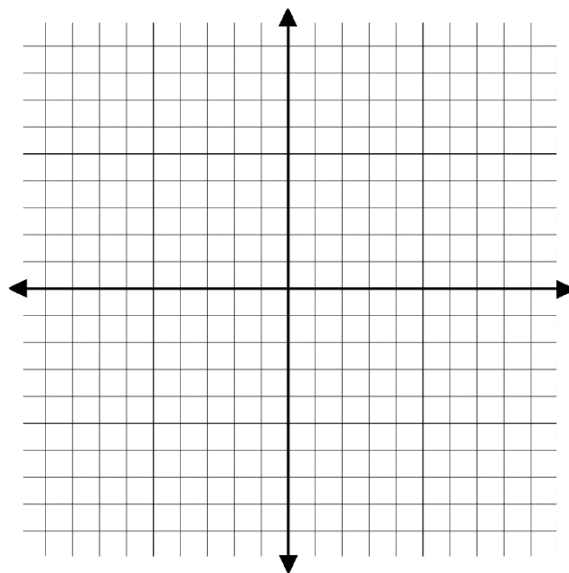
Plot, Sketch, Dot, Label

Your Turn:

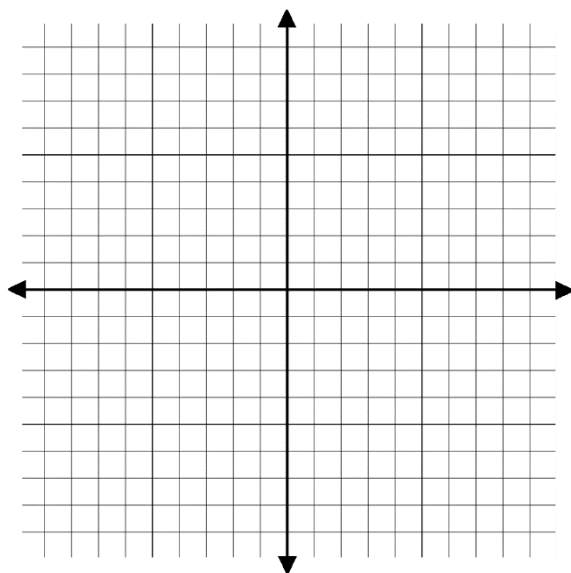
1. $f(x) = x^2 - 2x - 3$



2. $f(x) = 2x^2 + 3x - 5$



3. $f(x) = 5 - 4x - x^2$



Homework:

Find the coordinates of the vertex, the intercepts, and sketch a graph.

1) $f(x) = -3(x - 2)^2 + 12$

2) $f(x) = -2(x + 4)^2 - 8$

3) $f(x) = 3x^2 - 12x + 1$

4) $f(x) = -2x^2 + 8x - 1$

5) $f(x) = 2x - x^2 + 3$

6) $f(x) = 2x^2 - 7x - 4$

Chapter 2: Polynomial and Rational Functions
Topic 2: Quadratic Functions (Day 2)

Do Now: A parabola has a minimum or maximum point at its vertex. If a parabola opens up, its vertex is a minimum. If a parabola opens down, its vertex is a maximum. For each parabola below, state if it has a min or a max, and determine the coordinate point.

1. $f(x) = 3x^2 - 12x - 1$

2. $f(x) = -4x^2 - 3 + 8x$

3. $f(x) = 5x - 5x^2$

4. $f(x) = 6 - 4x + x^2$

Applying Quadratic Functions: Minimum & Maximum Values

Example: The function below models the number of people, $f(x)$, in millions, receiving food stamps x years after 1990: $f(x) = -0.5x^2 + 4x + 19$

(a) In which year was this number at its maximum?

(b) How many food stamp recipients were there for that year?

Full sentence conclusion:

Example: The function below models the number of accidents, $f(x)$, per 50 million miles driven, in terms of a driver's age, x , for drivers between the ages of 16 and 74 years of age: $f(x) = 0.4x^2 - 36x + 1000$

(a) What is the age of a driver having the least number of car accidents?

(b) What is the minimum number of car accidents per 50 million miles driven?

Full sentence conclusion:

Applying Quadratic Functions: Minimizing or Maximizing conditions

For each example below, we will need to interpret what each question is trying to accomplish, create equations which fit that goal, and use the equations together to create a quadratic function.

We will follow 5 steps for these questions:

Step 1: Identify what must be maximized or minimized as an equation.

Step 2: Identify what the conditions of the question are as an equation.

Step 3: Rewrite the max/min equation using ONE variable (ideally, x) and put in standard form.

Hint: To rewrite as x , substitute out y

Step 4: Find the max/min of that equation.

Step 5: Answer the question(s) posed in the problem using the condition from step 2.

Type 1: Solving a Number Problem

Example: Among all pairs of numbers whose sum is 40, mathematically find a pair whose product is as large as possible. What is the maximum possible product?

Step 1: What must be maximized?

The product of two numbers:

Step 2: What are the conditions of the question?

The sum of the numbers must be 40:

Step 3: Rewrite the maximization equation with one variable, in standard form.

Step 4: Find the maximum value of the equation.

Step 5: Answer the question(s) posed in the problem using the condition from step 2.

Example: Among all pairs of numbers whose sum is 28.5, mathematically find a pair whose product is as large as possible. What is the maximum possible product?

Type 2: Maximizing Area

Example: You have 100 yards of fencing to enclose a rectangular region to keep horses. Find the dimensions of the rectangle that maximize the enclosed area. What is the maximum area?

Step 1: What must be maximized?

The area of the region:

Step 2: What are the conditions of the question?

The perimeter is 100 yards:

Step 3: Rewrite the maximization equation with one variable, in standard form.

Step 4: Find the maximum value of the equation.

Step 5: Answer the question(s) posed in the problem using the condition from step 2.

Example: You have 320 feet of fencing to enclose a rectangular region for farming. One side of the region is a river which does NOT need fencing. Find the dimensions of the rectangle that maximize the enclosed area. What is the maximum area?

Example: A rectangular playground is to be fenced off and divided into two by another parallel fence inside. 600 feet of fencing is used. Find the dimensions of the playground that maximize the total enclosed area. What is the maximum area?

Homework:

1)

When the shot whose path is shown by the red graph is released at an angle of 65° , its height, $g(x)$, in feet, can be modeled by

$$g(x) = -0.04x^2 + 2.1x + 6.1,$$

where x is the shot's horizontal distance, in feet, from its point of release. Use this model to solve parts (a) through (c) and verify your answers using the red graph.

- a. What is the maximum height, to the nearest tenth of a foot, of the shot and how far from its point of release does this occur?
- b. What is the shot's maximum horizontal distance, to the nearest tenth of a foot, or the distance of the throw?
- c. From what height was the shot released?

2)

Among all pairs of numbers whose difference is 24, find a pair whose product is as small as possible. What is the minimum product?

3)

Among all pairs of numbers whose sum is 20, find a pair whose product is as large as possible. What is the maximum product?

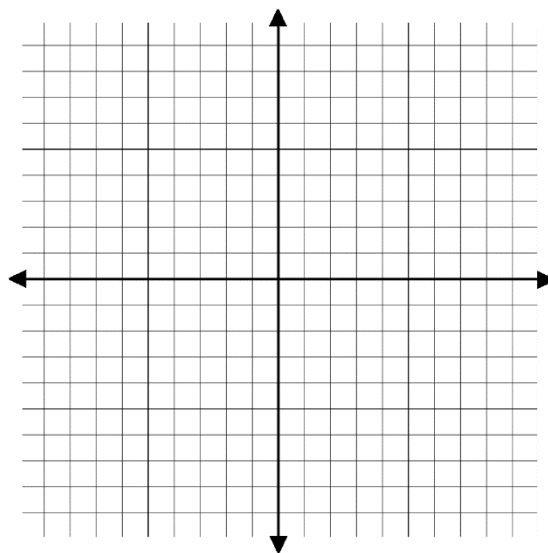
4)

Among all pairs of numbers whose difference is 16, find a pair whose product is as small as possible. What is the minimum product?

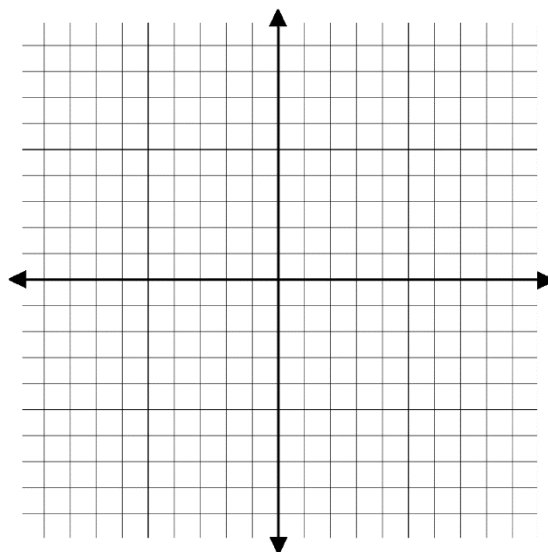
Chapter 2: Polynomial and Rational Functions**Topic 2: Quadratic Functions (Day 3)****Extra Practice****Part I: Graphing parabolas from key features.**

For each example below, graph the parabola using four key features (up/down, vertex, x-intercepts, y-intercept). Change the scale of the graph if necessary

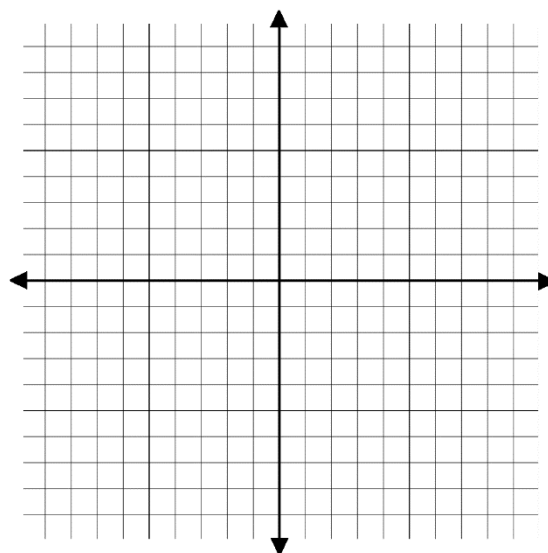
1. $f(x) = 2x^2 + 8x + 16$



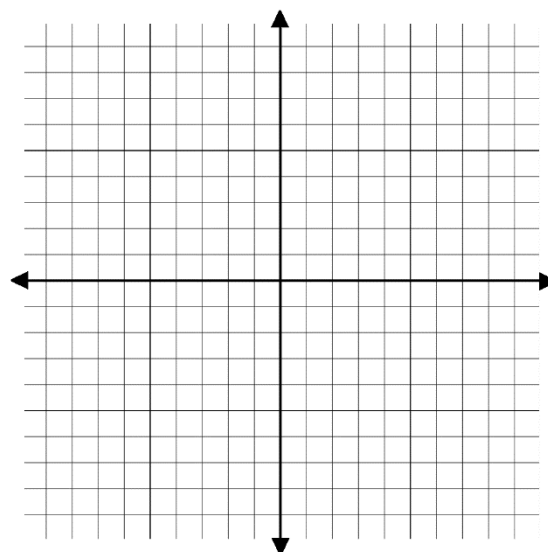
2. $f(x) = -(x - 1)^2 + 10$



3. $f(x) = 2x^2 - 4$

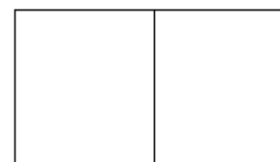


4. $f(x) = \frac{1}{2}(x + 5)^2 - 8$



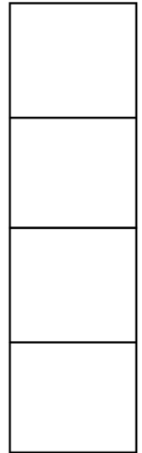
Part II: Applied quadratic functions.

5. A farmer has 120 feet of fencing with which to enclose two adjacent rectangular pens as shown. What dimensions should be used so that the enclosed area will be maximized? What will that area be?



6. Find two positive numbers such that the sum of the first plus five times the second is 50, and their product is a maximum.

7. A rancher has 180 feet of fencing with which to enclose four adjacent rectangular corrals as shown. What dimensions should be used so the enclosed area will be maximized? What will that area be?



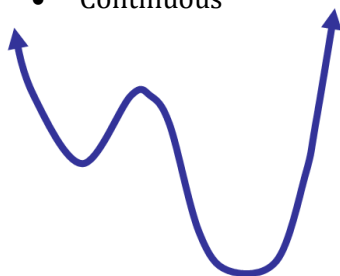
Chapter 2: Polynomial and Rational Functions
Topic 3: Polynomial Functions and Their Graphs

What does/doesn't a polynomial function graph look like?

Polynomial functions of any degree (linear, quadratic, or higher-degree) must have graphs that are smooth and continuous. There can be no sharp corners on the graph. There can be no breaks in the graph; you should be able to sketch the entire graph without picking up your pencil.

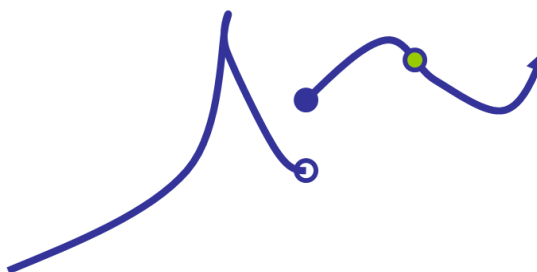
Polynomial function

- Smooth, rounded turns
- Continuous



Not a polynomial function

- Sharp Turn
- Discontinuous



End Behavior

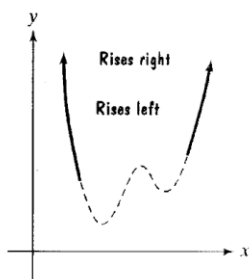
The behavior of the graph of a function to the far left or far right is called the end behavior. Although the graph of a polynomial function may have intervals where it increases or decreases, the graph will eventually continue to positive or negative infinity on both ends, without bound, as it rises or falls.

General Guidelines:

When the highest power is EVEN:

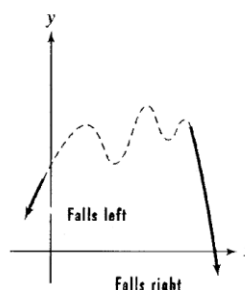
With a positive coefficient:

If the leading coefficient is positive, the graph rises to the left and to the right.



With a negative coefficient:

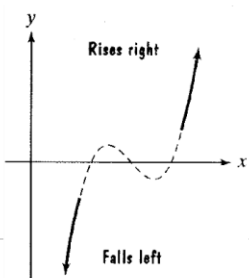
If the leading coefficient is negative, the graph falls to the left and to the right.



When the highest power is ODD:

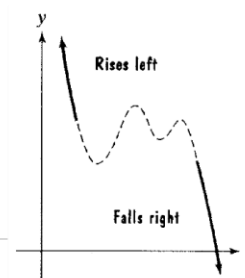
With a positive coefficient:

If the leading coefficient is positive, the graph falls to the left and rises to the right.



With a negative coefficient:

If the leading coefficient is negative, the graph rises to the left and falls to the right.



Zeros of a Polynomial Function: $f(x) = 0$

Recall: The highest degree of the equation will indicate how many roots the equation has.

Set the equation equal to zero and solve by factoring. These are the point where the graph interacts with the x-axis. Typically, the graph goes directly through the x-axis at these roots:

Examples: Find all zeros of the functions

$$f(x) = x^3 + 2x^2 - 9x - 18$$

$$f(x) = -x^4 + 4x^3 - 4x^2$$

$$f(x) = x^3 + 3x^2 - x - 3$$

$$f(x) = x^3 - 2x^2$$

Note that ALL zeroes of functions can be CHECKED by GRAPHING

Multiplicity - the number of times a root is associated with an equation. When a root has a multiplicity (more than one) the graph CURVES at the x-axis

All of the examples below have a single root at -3 . How many times does the root repeat for each?

$$f(x) = (x + 3)^2$$

$$f(x) = (x + 3)(x + 3)$$

$$f(x) = (x + 3)^3$$

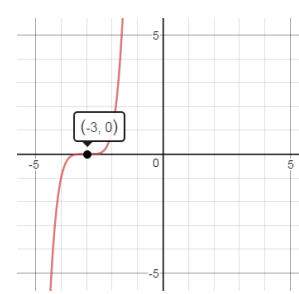
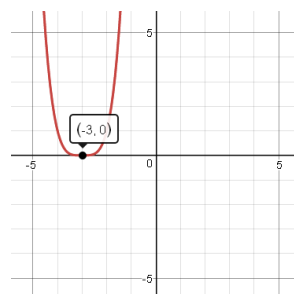
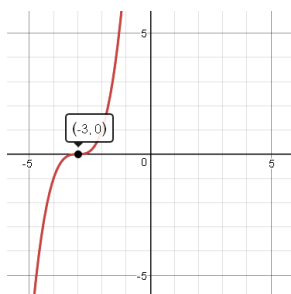
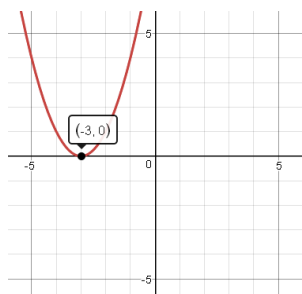
$$f(x) = (x + 3)(x + 3)(x + 3)$$

$$f(x) = (x + 3)^4$$

$$f(x) = (x + 3)(x + 3)(x + 3)(x + 3)$$

$$f(x) = (x + 3)^5$$

$$f(x) = (x + 3)(x + 3)(x + 3)(x + 3)(x + 3)$$



Make a conclusion:

When a root is repeated an ODD number of times:

When a root is repeated an EVEN number of times:

Turning Points:

As a general rule, a polynomial with highest degree n will have $n-1$ turning points on its graph.
This rule does not hold true when there is multiplicity.

Graphing Polynomial Functions

To graph polynomial functions we look for 4 key features:

1. End Behavior

- An ODD highest power: Ends are in opposite directions
 - With a positive coefficient: Rise Right, Fall Left
 - With a negative coefficient: Rise Left, Fall Right
- An EVEN highest power: Ends are in the same direction
 1. With a positive coefficient: Rises
 2. With a negative coefficient: Falls

2. What are the x-intercepts?

- Solve by setting the equation equal to zero ($f(x) = 0$)
 1. Repeated roots: The graph touches the x-axis and turns around

3. What is the y-intercept?

- Solve by evaluating at zero ($f(0) =$)

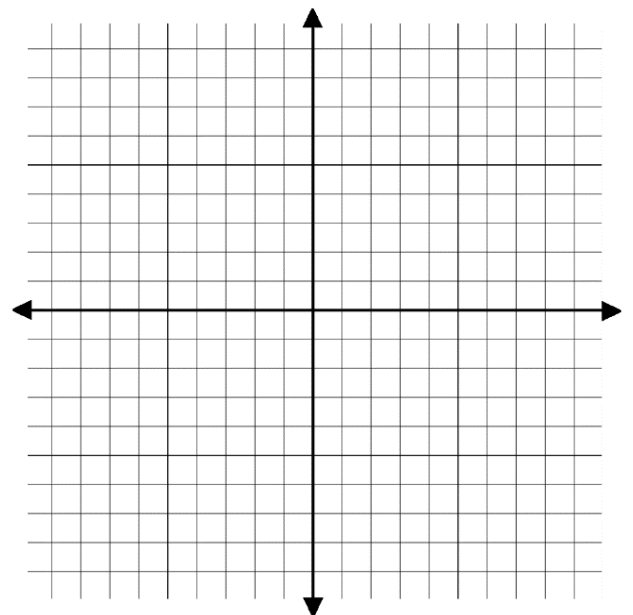
4. Turning Points

- The max number of turning points is one-less-than the highest power.

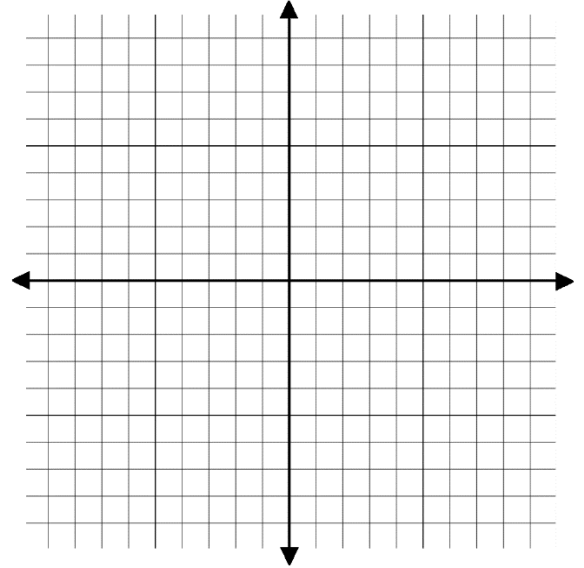
Then, plot all of the key points, and sketch a smooth polynomial.

Examples:

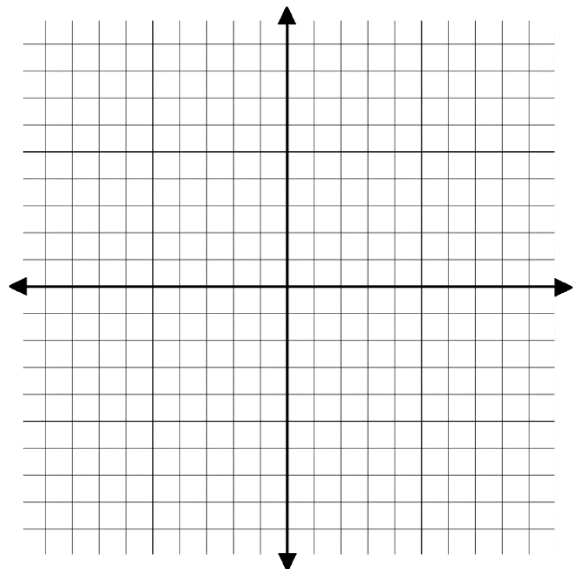
1. $f(x) = x^3 + 3x^2 - x - 3$



2. $f(x) = x^4 - 2x^2 + 1$



3. $f(x) = x^3 - 3x^2$



Homework:

- a. Use the Leading Coefficient Test to determine the graph's end behavior.
- b. Find the x -intercepts. State whether the graph crosses the x -axis, or touches the x -axis and turns around, at each intercept.
- c. Find the y -intercept.
- d. Determine whether the graph has y -axis symmetry, origin symmetry, or neither.
- e. If necessary, find a few additional points and graph the function. Use the maximum number of turning points to check whether it is drawn correctly.

1) $f(x) = x^3 + 2x^2 - x - 2$

2) $f(x) = x^3 + x^2 - 4x - 4$

3) $f(x) = -x^4 + 16x^2$

4) $f(x) = 6x - x^3 - x^5$

5) $f(x) = 3x^2 - x^3$

6) $f(x) = x^4 - 6x^3 + 9x^2$

Name: _____ Date: _____ Period: _____

Chapter 2: Polynomial and Rational Functions
Topic 4: Polynomial Division

Do Now: Long Division. Don't create a decimal; leave your remainder as "R".

$$17 \overline{)5208}$$

$$8 \overline{)98062}$$

Long Division of Polynomials

Always ensure that your dividend and divisor are in **descending** order!

When the divisor is a polynomial (usually a binomial) we only consider the first term when dividing.

Then, just like integer division:

- **Divide into the first term of the dividend, Multiply around, Subtract, Bring down. Repeat.**
- **Rewrite the expression as divisor · quotient + remainder**

1. $x + 3 \overline{)x^2 + 10x + 21}$

2. Divide $x^2 + 14x + 45$ by $x + 9$

3. $3x - 2 \overline{) 6x^3 - x^2 - 5x + 4}$

4. Divide $7 - 11x - 3x^2 + 2x^3$ by $x - 3$

5. Divide $6x^4 + 5x^3 + 3x - 5$ by $3x^2 - 2x$

Synthetic Division

We can only use synthetic division if:

- The divisor is of the form $x - c$
- In descending order, all terms must appear

Class example:

$x^3 + 4x^2 - 5x + 5$ divided by $x - 3$

1. Identify c : $c =$
2. Write in synthetic form:
 - Write c for the divisor
 - Write the coefficients of the dividend

$$\begin{array}{r|rrrr} 3 & 1 & 4 & -5 & 5 \\ \hline \end{array}$$

3. Carry down the leading coefficient
4. Multiply c by the bottom value, and carry it to the next row
5. ADD
6. Repeat steps 4 & 5
7. The final bottom value is the remainder
8. Using the solution line, re-write the quotient and remainder.
The highest degree of the quotient is one-less-than the dividend

Divide $5x^3 + 6x + 8$ by $x + 2$ using synthetic division

Divide $x^3 - 7x - 6$ by $x + 2$ using synthetic division

Remainder Theorem

Consider how we wrote these solutions in the form

$$\text{Equation} = \text{Dividend} \cdot \text{Quotient} + \text{Remainder}$$

$$f(x) = (x - c) \cdot q(x) + r$$

Evaluate this form at $f(c)$:

The Remainder Theorem

If a polynomial $f(x)$ is divided by $x - c$, then $f(c) = \text{remainder}$

Example using the remainder theorem:

Given $f(x) = x^3 - 4x^2 + 5x + 3$, use the remainder theorem to find $f(2)$

Verify by evaluating $f(2) =$

Take the remainder theorem a step further, and we get...

The Factor Theorem

Since $f(c) = \text{Remainder}$,

- If $f(c) = 0$, the remainder theorem tells us that the remainder is zero.
- If the binomial $x - c$ divides evenly into the polynomial without remainder, it must be a FACTOR of the polynomial

The Factor Theorem

If $f(c) = 0$, then $x - c$ is a factor of $f(x)$

Examples using the factor theorem:

Solve the equation $2x^3 - 3x^2 - 11x + 6 = 0$ given that 3 is a zero of the function.

From the question, we know that $f(3) = 0$. The factor theorem then tells us that $(x - 3)$ is a factor.

- Use synthetic division to divide.
- Write the equation in factored form using the result from synthetic division
- Finish factoring
- Solve

Solve the equation $15x^3 + 14x^2 - 3x - 2 = 0$ given that -1 is a zero of the function.

Solve the equation $2x^3 - 11x^2 + 7x - 5 = 0$ given that 4 is a zero of the function.

Name: _____

Date: _____

Period: _____

Homework:

Divide using long division:

1) $(6x^3 + 17x^2 + 27x + 20) \div (3x + 4)$ 2) $\frac{6x^3 + 13x^2 - 11x - 15}{3x^2 - x - 3}$

Divide using synthetic division:

3) $(5x^2 - 12x - 8) \div (x + 3)$ 4) $(x^5 + 4x^4 - 3x^2 + 2x + 3) \div (x - 3)$

Use the remainder theorem and synthetic division.

5) $f(x) = 2x^3 - 11x^2 + 7x - 5$

Chapter 2: Polynomial and Rational Functions
Topic 5: Zeros of Polynomial Functions (Day 1)

Recall: a 'zero' of a function is the values where the graph crosses the x-axis. These can also be called:

The Rational Zero Theorem

$$\text{Possible Rational Zeros} = \frac{\text{factors of the constant term}}{\text{factors of the leading coefficient}}$$

Notice that this rule uses the word "possible"!

In order to use the theorem, we list out all of the integers that are factors of the constant term and the factors of the leading coefficient. Since the rule uses integers (not just whole numbers), remember to include \pm on all of the factors.

Example #1: List all **possible** rational zeroes of $f(x) = -x^4 + 3x^2 + 4$

Factors of the constant term:

Factors of the leading coefficient:

Possible Rational Zeroes =

Split up each possible option and reduce in order to answer the question.

2. List all possible rational zeroes of $f(x) = x^3 + 2x^2 - 5x - 6$

3. List all possible rational zeroes of $f(x) = 15x^3 + 14x^2 - 3x - 2$

4. List all possible rational zeroes of $f(x) = 4x^5 + 12x^4 - x - 3$

Finding Actual Zeros of a Polynomial using the Rational Zero Theorem

Once we find a list of all possible rational zeros, we will begin testing values using the factor theorem (if $f(c) = 0$, meaning the synthetic division leaves no remainder, then it must be a zero of the equation). Once we have found one that is correct, the rest we will figure out with factoring and solving.

Example #1: Find all rational zeros of $f(x) = x^3 + 2x^2 - 5x - 6$
or: Solve $x^3 + 2x^2 - 5x - 6 = 0$

Possible rational zeros:

Using synthetic division, start testing until we find a zero of the polynomial:

The zero remainder when we test ___ tells us that it is a zero of $x^3 + 2x^2 - 5x - 6 = 0$

Rewrite, finish factoring, and solve.

2. Solve: $x^3 + 8x^2 + 11x - 20 = 0$

3. Solve: $f(x) = x^3 + 7x^2 + 11x - 3$

4. Solve: $f(x) = x^3 + x^2 - 5x - 2$

5. Solve: $f(x) = x^4 - 6x^2 - 8x + 24$

Name: _____

Date: _____

Period: _____

Homework:

- a. List all possible rational zeros.
- b. Use synthetic division to test the possible rational zeros and find an actual zero.
- c. Use the quotient from part (b) to find the remaining zeros of the polynomial function.

1) $f(x) = x^3 + x^2 - 4x - 4$

2) $f(x) = x^3 - 2x^2 - 11x + 12$

3) $f(x) = 2x^3 - 3x^2 - 11x + 6$

4) $f(x) = 2x^3 - 5x^2 + x + 2$

Chapter 2: Polynomial and Rational Functions
Topic 5: Zeros of Polynomial Functions (Day 2)

Working Backwards: Finding Polynomial Function with Given Zeros

Quickly, find the solutions to the following equations. Think GCF first:

(a) $2x^2 - 8 = 0$

(b) $x^2 - 4 = 0$

A quick reminder that when a GCF is a constant number (no variable) it doesn't appear in the solution set.

Because of that, when we work backwards in this section, we will always need to consider that there *was* a constant GCF in the function.

Example #1: Find a fourth-degree polynomial function $f(x)$ with real coefficients that has ± 2 and $\pm i$ as zeros, and that $f(3) = -150$

We will start with a blank form. Since this is the fourth-degree, we will account for 4 roots:

$$f(x) = a(x - c_1)(x - c_2)(x - c_3)(x - c_4)$$

Start with the zeros and simplify as much as possible:

Use the given $f(3) = -150$ to solve for a :

Write the final polynomial function:

2. Find a third-degree polynomial function $f(x)$ with real coefficients that has -3 and i as zeros and such that $f(1) = 8$
Hint: If i is a zero, what else MUST be a zero as well?

3. Find a third degree polynomial function $f(x)$ with real coefficients that has 1 and $5i$ as zeroes and such that $f(-1) = -104$

BONUS: Find a fourth-degree polynomial function $f(x)$ with real coefficients that has -2 , 5 and $3 + 2i$ as zeros and such that $f(1) = -96$

Hint: If $3 + 2i$ is a zero, what else MUST be a zero as well?

Name: _____ Date: _____ Period: _____

Homework:

Find an n th degree polynomial with real coefficients satisfying the given conditions.

1) $n = 3$; 1 and $5i$ are zeros; $f(-1) = -104$

2) $n = 3$; 4 and $2i$ are zeros; $f(-1) = -50$

3) $n = 3$; -5 and $4 + 3i$ are zeros; $f(2) = 91$

Chapter 2: Polynomial and Rational Functions
Topic 6: Rational Functions & Their Graphs (Day 1)

Rational functions, like rational numbers, will involve a fraction. We will discuss rational functions in the form:

$$f(x) = \frac{p(x)}{q(x)}$$

where $p(x)$ and $q(x)$ are polynomial functions, and $q(x) \neq 0$

Domains of Rational Functions:

Typically, functions have unlimited domain values, $-\infty < x < \infty$. However, remember that functions are graphed on the real coordinate plane. Because of this, there are two important restrictions we must remember.

- (1) A denominator of a fraction can never equal zero. When functions contain a fraction, we must limit the domain in order to eliminate any value(s) which cause the denominator to be zero.

Example: What is the domain of $f(x) = \frac{x^3}{x^2-9}$

- (2) We cannot take the square root of a negative number. When a function contains a radical, we must limit the domain in order to ensure the radicand is not negative.

Example: What is the domain of $g(x) = 2\sqrt{5x-15}$

A basic introduction to limits

The reciprocal function is defined as $f(x) = \frac{1}{x}$. What is the domain?

View the graph on your calculator

Because of this domain limitation, we see that the graph behaves strangely at the line $x = 0$.

Create a chart of values approaching $x = 0$ from the left:

x	-1	-0.5	-0.1	-0.01	-0.001
$f(x)$					

Make a conclusion: As x approaches zero from the left, $f(x)$ approaches _____

Written in arrow notation: As $x \rightarrow 0^-$, $f(x) \rightarrow -\infty$

Create a chart of values approaching $x = 0$ from the right:

x	0.001	0.01	0.1	0.5	1
$f(x)$					

Make a conclusion: As x approaches zero from the right, $f(x)$ approaches _____

Written in arrow notation: As $x \rightarrow 0^+$, $f(x) \rightarrow \infty$

Additionally, the graph seems to 'level out' as it goes out to infinity in both directions.

Create a chart of values as x approaches negative infinity:

x	-1	-10	-100	-1000
$f(x)$				

Make a conclusion: As x approaches negative infinity, $f(x)$ approaches _____

Written in arrow notation: As $x \rightarrow -\infty$, $f(x) \rightarrow 0$

Create a chart of values as x approaches infinity:

x	1	10	100	1000
$f(x)$				

Make a conclusion: As x approaches infinity, $f(x)$ approaches _____

Written in arrow notation: As $x \rightarrow \infty$, $f(x) \rightarrow 0$

Vertical Asymptotes of Rational Functions



Recall that an asymptote of a graph is a line that the graph is infinitely approaches but never touches. In the example above of $\frac{1}{x}$, the y-axis was a vertical asymptote.

Rational functions may or may not have vertical asymptotes, and could have more than one. These asymptotes will exist at any x-value which would cause the denominator of the function to equal zero; that is, these asymptotes will occur at the $x \neq$ values of the domain limitation.

To find the vertical asymptotes:

- Reduce the full function by factoring and canceling, if possible
- Evaluate the remaining denominator, $q(x) = 0$

Examples: Find the vertical asymptote(s), if any, of the following functions

1. $f(x) = \frac{x}{x^2-9}$

2. $f(x) = \frac{x+3}{x^2-9}$

3. $f(x) = \frac{x+3}{x^2+9}$

Horizontal Asymptotes of Rational Functions



In the example above of $\frac{1}{x}$, the x-axis was a horizontal asymptote.

Rational functions may or may not have horizontal asymptotes, but cannot have more than one.

To find the horizontal asymptote, we use the layout of the full function $f(x) = \frac{p(x)}{q(x)}$.

If	Then
Highest power of numerator < highest power of denominator	Horizontal Asymptote = x-axis
Highest power of numerator > highest power of denominator	No horizontal asymptote. Check for Slant Asymptote.
Highest power of numerator = highest power of denominator	Horizontal Asymptote = $\frac{\text{leading coefficient of numerator}}{\text{leading coefficient of denominator}}$

In less formal terms:

- *Smaller numerator power means x-axis*
- *Bigger numerator power means none*
- *When equal, divide the leading coefficients*

Examples: Find the horizontal asymptote, if any, of the following functions

1. $f(x) = \frac{4x}{2x^2+1}$

2. $f(x) = \frac{4x^2}{2x^2+1}$

3. $f(x) = \frac{4x^4}{2x^2+1}$

Slant Asymptotes



Slant asymptotes exist when the high-degree of the numerator is **exactly** one more than the high-degree of the denominator.

To find the equation of the slant asymptote, we perform the division of the function. The quotient becomes the equation of the slant asymptote.

Example: Find the slant asymptote of $f(x) = \frac{x^2 - 4x - 5}{x - 3}$

Homework:

Find any holes or asymptotes.

1) $f(x) = \frac{x}{x+4}$

2) $g(x) = \frac{x+3}{x(x+4)}$

3) $f(x) = \frac{x^2 - 25}{x - 5}$

4) $h(x) = \frac{x+6}{x^2 + 2x - 24}$

5) $r(x) = \frac{x^2 + 2x - 24}{x+6}$

6) $h(x) = \frac{x}{x(x-3)}$

7) $f(x) = \frac{15x}{3x^2 + 1}$

8) $g(x) = \frac{15x^2}{3x^2 + 1}$

9) $h(x) = \frac{15x^3}{3x^2 + 1}$

Chapter 2: Polynomial and Rational Functions
Topic 6: Rational Functions & Their Graphs (Day 2)

Graphing Rational Functions

To graph polynomial functions we look for 4 key features:

5. **Find the y-intercept (if there is one)**
 - Solve by evaluating at zero ($f(0) =$)
6. **Find the x-intercepts (if there are any)**
 - Solve by setting the NUMERATOR of the equation equal to zero ($p(x) = 0$)
7. **Find any vertical asymptotes**
 - Solve by setting the DENOMINATOR of the equation equal to zero ($q(x) = 0$)
8. **Find any horizontal asymptote**
 - Rules laid out above
 - If there's no horizontal, check for slant

Plot and sketch the above features.

- Asymptotes must be drawn as dotted lines.

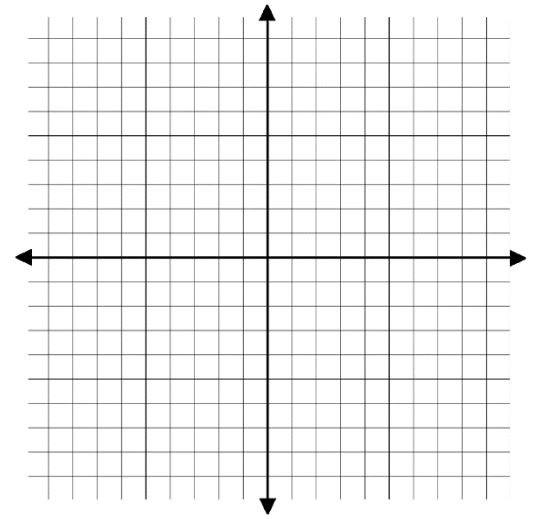
Add a few points in between these features.

Sketch smooth curves.

Example #1: Graph $f(x) = \frac{2x}{x-1}$

- y-intercept:
- x-intercept(s):

- Vertical Asymptotes:



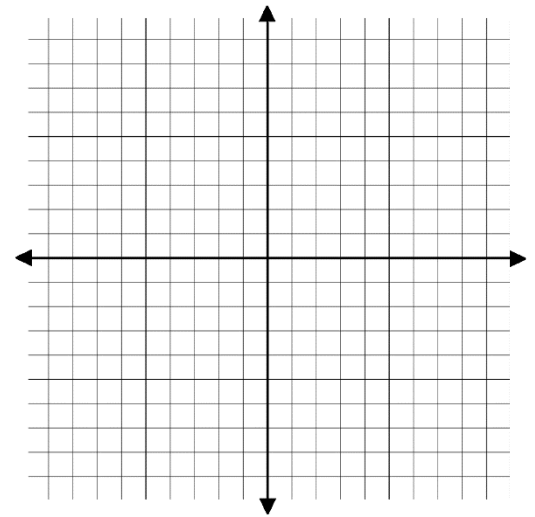
- Horizontal Asymptotes:

Plot & sketch the above features

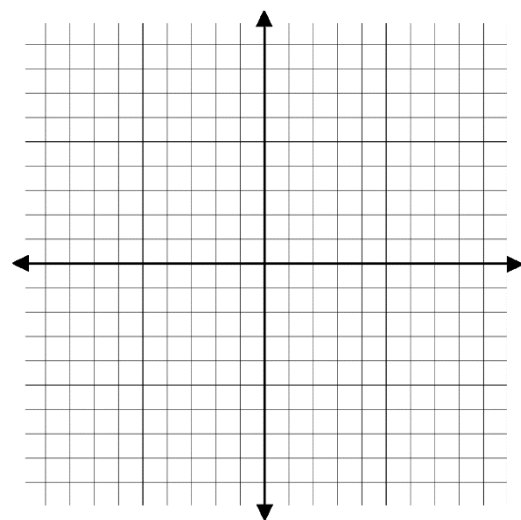
Add a few points

Sketch smooth curves

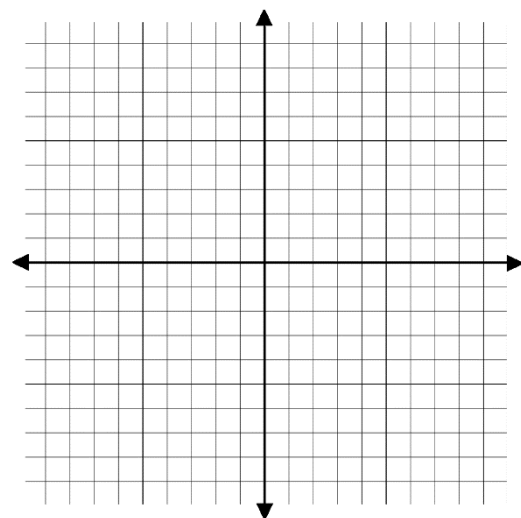
Example #2: Graph $f(x) = \frac{3x^2}{x^2-4}$



Example #3: Graph $f(x) = \frac{x^4}{x^2+1}$



Example #4: Graph $f(x) = \frac{x^2+1}{x-1}$



Name: _____

Date: _____

Period: _____

Homework

Graph the functions.

1) $f(x) = \frac{3x}{x-1}$

2) $f(x) = \frac{4x}{x^2-1}$

3) $f(x) = \frac{4x^2}{x^2-9}$

4) $f(x) = \frac{-3x}{x+2}$

5) $f(x) = -\frac{2}{x^2-1}$

6) $f(x) = \frac{-2}{x^2-x-2}$

Chapter 2: Polynomial and Rational Functions
Topic 7: Polynomial and Rational Inequalities

A review from Algebra 2: Solving Polynomial Inequalities

- Express the inequality to zero.
- Solve for all zeros of the function; these are your boundary points.
- Plot boundary points. Choose and evaluate test points.
- Express your answer on the graph and in set notation.

Example #1: Solve and graph the solution set of $2x^2 + x > 15$

#2: Solve and graph the solution set of $x^2 - x \geq 20$

#3: Solve and graph the solution set of $x^3 + x^2 \leq 4x + 4$

Solving Rational Inequalities

DO NOT CROSS MULTIPLY!!!

- Express the inequality to zero.
- Solve for all zeros of the function by setting the numerator AND denominator to zero; these are your boundary points.
- Plot boundary points. Choose and evaluate test points.
- Express your answer on the graph and in set notation.

Example #4: Solve and graph the solution set of $\frac{3x+3}{2x+4} > 0$

#5: Solve and graph the solution set of $\frac{x+1}{x+3} \geq 2$

#6: Solve and graph the solution set of $\frac{2x}{x+1} \geq 1$

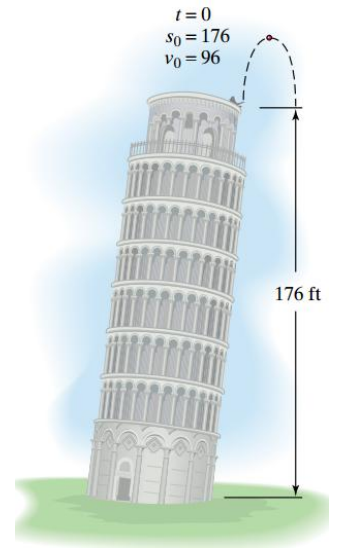
Applications

Many daily occurrences can be modeled with math, especially expressed as functions. For example, an item thrown upwards, eventually returning downwards can be modeled with a quadratic function. Since the rate of gravity is constant, this can always be modeled using the following:

$$s(t) = -16t^2 + v_0t + s_0$$

Showing the height is a function of time, where v_0 is the original velocity and s_0 is the original height

Example #7: A ball is thrown vertically upward from the top of the Leaning Tower of Pisa (176 feet high) with an initial velocity of 96 feet per second. During which time period will the ball's height exceed that of the tower?



#8: A toy rocket is propelled straight up from the ground level with an initial velocity of 80 feet per second. During which time period will the rocket be more than 64 feet above the ground?

Name: _____

Date: _____

Period: _____

Homework:

Solve.

1) $x^2 - 5x + 4 > 0$

2) $9x^2 + 3x - 2 \geq 0$

3) $\frac{x - 4}{x + 3} > 0$

4) $\frac{x + 5}{x + 2} < 0$

5) $\frac{4 - 2x}{3x + 4} \leq 0$

6) $\frac{3x + 5}{6 - 2x} \geq 0$

Name: _____ Date: _____ Period: _____

Chapter 2: Polynomial and Rational Functions
Topic 8: Modeling Using Variation

Direct Variation

As goes x, so goes y

If a situation is described by an equation in the form $y = kx$, we say that **y varies directly as x** or **y is directly proportional to x**. The number, **k**, is called the **constant of variation**.

To solve these problems, we have two options.

Option #1: Use the given information to solve for k and create a general equation. Use that equation to solve the question posed.

Option #2: Using the equation given above, solve for k. $k = \frac{y}{x}$. Since k is constant for the situation, all values of y and x that fit the given scenario will be equally proportional in that ratio. We can set up the proportion $\frac{y}{x} = \frac{y}{x}$ to solve the question posed.

Example 1: The amount of garbage, G, varies directly as the population, p. Allegheny County PA has a population of 1.3 million people and creates 26 million pound of garbage each week. Find the weekly garbage produced by NYC with a population of 7.3 million

Solve by option #1:

Solve by option #2:

Example 2: The pressure of water, W, on an object below the surface varies directly as its distance, d, below the surface. If a submarine experiences a pressure of 25 pounds per square inches 60 feet below the surface, how much pressure will it experience 330 feet below the surface?

Solve by whichever option you prefer

We can also encounter situations where the x-value (independent variable) is raised to a higher power. This does not change the basics of the equation or the procedures needed to solve. The equation then becomes $y = kx^n$, where **y varies directly as the nth power of x**.

When there is a higher power for x, we must take the time to write the equation first with the appropriate exponent. Then, we can choose again between option #1 or option #2 from above.

Example 3: The distance, S, that a body falls from rest varies directly as the square of the time, t, of the fall. If skydivers fall 64 feet in 2 seconds, how far will they fall in 4.5 seconds?

Equation:

Solve using option #1:

Solve using option #2:

Example 4: The distance required to stop a car varies directly as the square of its speed. If 200 feet are required to stop a car traveling 60 miles per hour, how many feet are required to stop a car traveling 100 miles per hour?

Solve by whichever option you prefer

Inverse Variation (indirect variation)

As goes x, inversely goes y

If a situation is described by an equation in the form $y = \frac{k}{x}$, we say that **y varies indirectly as x** or **y is inversely proportional to x**. The number, **k**, is called the **constant of variation**.

To solve these problems, we have two options.

Option #1: Use the given information to solve for k and create a general equation. Use that equation to solve the question posed.

Option #2: Using the equation given above, solve for k. $k = yx$. Since k is constant for the situation, all values of y and x that fit the given scenario will be inversely proportional of the same product. We can set up the proportion $yx = yx$ to solve the question posed.

Example 5: The pressure, P, of the gas in a spray can varies inversely as the volume, V, of the container's substance. The pressure of the gas in a container, whose volume is 8 cubic inches, is 12 pounds per square inch. If the sample expands to a volume of 22 cubic inches, what is the new pressure of the gas?

Solve by option #1:

Solve by option #2:

Example 6: The price, P, of oil varies inversely as the supply, S. An OPEC nation sells oil for \$1950 per barrel when its daily production level is 4 million barrels. At what price will it sell oil if the daily production level is decreased to 3 million barrels??

Solve by whichever option you prefer

Name: _____

Date: _____

Period: _____

Homework:

y varies directly as x . $y = 65$ when $x = 5$. Find y when
1) $x = 12$.

y varies inversely as x . $y = 12$ when $x = 5$. Find y when
2) $x = 2$.

3)

An object's weight on the Moon, M , varies directly as its weight on Earth, E . Neil Armstrong, the first person to step on the Moon on July 20, 1969, weighed 360 pounds on Earth (with all of his equipment on) and 60 pounds on the Moon. What is the Moon weight of a person who weighs 186 pounds on Earth?

4)

The distance that a spring will stretch varies directly as the force applied to the spring. A force of 12 pounds is needed to stretch a spring 9 inches. What force is required to stretch the spring 15 inches?

5)

The water temperature of the Pacific Ocean varies inversely as the water's depth. At a depth of 1000 meters, the water temperature is 4.4° Celsius. What is the water temperature at a depth of 5000 meters?